

## Estimate Microstrip Substrate Relative Dielectric Constant

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These equations can be applied to a number of different substrate materials to estimate the relative dielectric constant of circuit boards with microstrip transmission lines.

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Microstrip lines are commonly used in microwave circuits, although their performance is subject to the type of substrate material selected for a printed-circuit board (PCB). One of the more important material parameters for the substrate is the relative dielectric constant, which should be known before designing a high-frequency, microstrip-based circuit. What follows is a simple and practical method for estimating the relative dielectric constant of a microstrip substrate based on well-known microstrip line empirical equations.

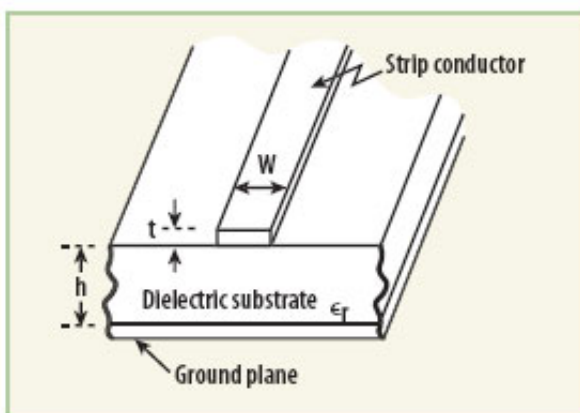
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1. Microstrip transmission lines consist of a strip conductor and a ground metal plane separated by a dielectric medium.

Microstrip transmission lines consist of a strip conductor and a ground metal plane separated by a dielectric medium ([Fig. 1](#)). The dielectric material serves as a substrate and it is sandwiched between the strip conductor and the ground plane. Typical substrate materials include alumina, silicon, and polytetrafluoroethylene (PTFE). Important physical parameters of the microstrip line include the line width,  $W$ , substrate height,  $h$ , strip conductor thickness,  $t$ , and substrate relative dielectric constant,  $\epsilon_r$ . Important electrical parameters for microstrip line design are the characteristic impedance,  $Z_0$ , the guide wavelength,  $\lambda_g$ , and the attenuation constant,  $\alpha$ .

The electromagnetic (EM) field lines in the microstrip line are not contained entirely in the substrate but also propagate outside of the microstrip (Fig. 2). Therefore, the propagating mode in the microstrip line is not a pure transverse-electromagnetic (TEM) mode but, rather, a quasi-TEM. The phase velocity in the microstrip line is given by:

$$v_p = \frac{c}{\sqrt{\epsilon_{re}}} \quad (1)$$

where:

$c$  = the speed of light and  $\epsilon_{re}$  = the effective dielectric constant of the microstrip line.

The value of  $\epsilon_{re}$  is significantly lower than that of  $\epsilon_r$  when the fields external to the substrate are taken into account. The guide wavelength,  $\lambda_g$  is related to the free-space wavelength,  $\lambda$ , as shown by Eq. 2:

$$\lambda_g = \frac{v_p}{f} = \frac{c}{f\sqrt{\epsilon_{re}}} = \frac{\lambda}{\sqrt{\epsilon_{re}}} \quad (2)$$

Available numerical methods for the characterization of microstrip lines involve extensive computations. Closed-form expressions are necessary for the computer-aided design (CAD) and optimization of microstrip circuits. The closed-form expressions for  $Z_0$  and  $\epsilon_{re}$  apply to the following two sets of conditions<sup>2</sup>:

For  $W/h \leq 1$ ,

$$Z_0 = \frac{60}{\sqrt{\epsilon_{re}}} \ln(8h/W + 0.25W/h) \quad (3)$$

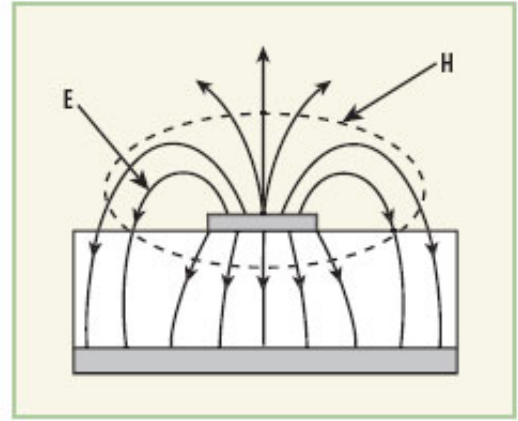
where:

$$\epsilon_{re} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[ (1 + 12h/W)^{-1/2} + 0.04(1 - W/h)^2 \right] \quad (4)$$

And for  $W/h \geq 1$ ,

$$Z_0 = \frac{120\pi / \sqrt{\epsilon_{re}}}{W/h + 1.393 + 0.667 \ln(W/h + 1.44)} \quad (5)$$

where:



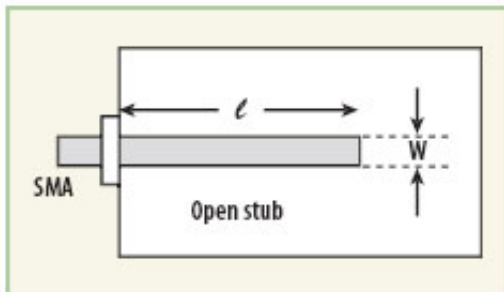
2. The EM fields are not contained entirely within a microstrip line but propagate outside of the line as well.

$$\epsilon_{re} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} (1 + 12h/W)^{-1/2} \quad (6)$$

These expressions provide accuracy of better than 2 percent.

## Strip thickness

The results discussed above assume a negligible strip thickness. In reality, the thickness of the strip has some bearing on the electrical performance, and the strip thickness,  $t$ , affects the microstrip characteristics. However, for a thickness- to-height ratio ( $t/h$ ) less than or equal to 0.005, the agreement between experimental and theoretical results obtained by assuming  $t/h = 0$  is acceptable.<sup>2</sup>



**3. This type of test fixture can be used for evaluating a microstrip circuit board under open-circuit conditions with a microwave vector network analyzer.**

Before commencing with a particular design or any use of design equations, a microwave circuit designer must select the PCB substrate material most appropriate for the required application. The specific high-frequency loss characteristics and other performance parameters will be part of that choice.

If the substrate thickness,  $h$ , of the circuit board is known, then the relative dielectric constant of the board can be determined using a simple experiment. This involves placing the test microstrip circuit board with specific length,  $l$ , and width,  $W$  (where  $W > h$ ) into a test fixture and evaluating the microstrip circuit board under open-circuit conditions over a specified range of frequencies. [Figure 3](#) offers an example of a test fixture for this

purpose. The input impedance (or reactance) of the line should be recorded for each sample frequency.

It is well known that the input admittance of an open-circuited stub is given by:

$$Y_{oc} = jY_o \tan\left(\frac{2\pi\ell}{\lambda}\right) \quad (7)$$

The characteristic impedance of the open-circuit microstrip line is:

$$Z_o = -X_{oc} \tan\left(\frac{2\pi\ell}{\lambda_g}\right) \quad (8)$$

where  $X_{oc}$  is the measured input reactance of the microstrip line at the specified frequency (assuming it to be lossless).

Substituting Eq. 2 in Eq. 8 yields:

$$Z_o = -X_{oc} \tan\left(\frac{2\pi\ell\sqrt{\epsilon_{re}}}{\lambda}\right) \quad (9)$$

where:

$$\lambda = c/f$$

For a microstrip line with  $W/h \geq 1$ ,  $Z_o$  is given by Eq. 5 and can be rewritten as:

$$Z_o = \frac{A}{\sqrt{\epsilon_{re}}} \quad (10)$$

and  $\epsilon_{re}$  is given by Eq. 6 as:

$$\epsilon_{re} = 0.5[\epsilon_r + 1 + B(\epsilon_r - 1)] \quad (11)$$

where:

$$A = 120\pi[W/h + 1.393 + 0.667 \ln(W/h + 1.444)]^{-1} \quad (12)$$

and

$$B = (1 + 12h/W)^{-1/2} \quad (13)$$

Substituting Eq. 11 into Eq. 10 and equating Eqs. 10 and 8 yields:

$$A[0.5(\epsilon_r + 1 + B(\epsilon_r - 1))]^{-1/2} = -X_{oc} \tan\left(\frac{2\pi\ell}{\lambda}[0.5(\epsilon_r + 1 + B(\epsilon_r - 1))]^{-1/2}\right) \quad (14)$$

To find  $\epsilon_r$ , Eq. 14 can be solved using the Newton-Raphson iterative formula. Thus, Eq. 14 can be formulated as:

$$f(\epsilon_r) = A[0.5(\epsilon_r + 1 + B(\epsilon_r - 1))]^{-1/2} + X_{oc} \tan\left(\frac{2\pi\ell}{\lambda}[0.5(\epsilon_r + 1 + B(\epsilon_r - 1))]^{1/2}\right) \quad (15)$$

A computer program can be written to minimize  $f(\epsilon_r)$  according to the algorithm:

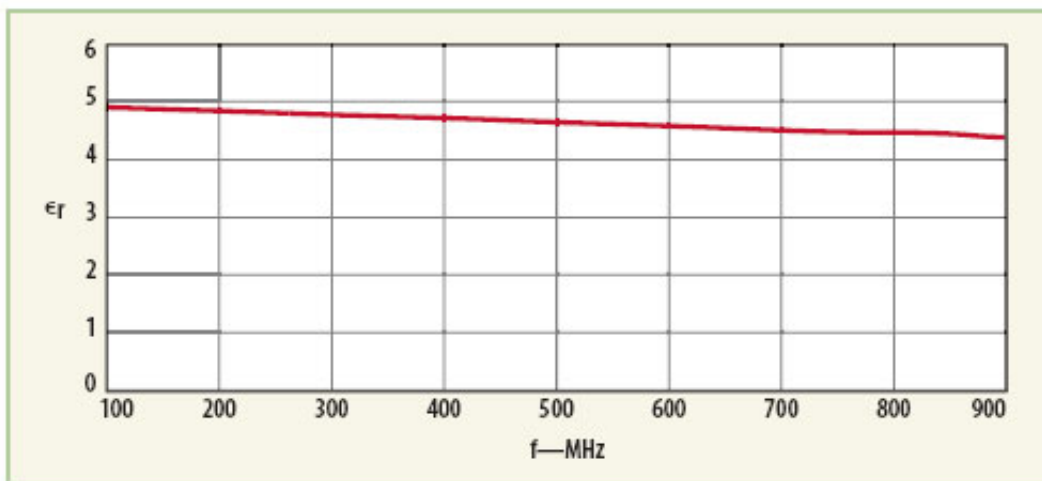
$$\epsilon_{r(i+1)} = \epsilon_{r(i)} - \frac{f(\epsilon_{r(i)})}{f'(\epsilon_{r(i)})} \quad (16)$$

where:

$$f'(\epsilon_r) = -0.5A\epsilon_{re}^{-3/2}(0.5(B+1)) + X_{oc} \sec^2\left(\frac{2\pi\ell}{\lambda}\epsilon_{re}^{-1/2}\right) \cdot \frac{\pi\ell}{\lambda}(0.5(B+1))\epsilon_{re}^{-1/2} \quad (17)$$

and where  $\epsilon_{re}$  is substituted from Eq. 11. An initial value of  $\epsilon_r$  close to the actual value is necessary for the algorithm to converge as quickly as possible.

This estimation procedure was used to determine the relative dielectric constant for two types of substrate materials: Teflon-fiber glass (PTFE) and epoxy-glass boards. The relative dielectric constant of the PTFE substrate was found to be 2.68. This value is approximately constant over a wide range of frequencies. The microstrip line for this type of substrates can be considered lossless at frequencies to several gigahertz.



4. These data show the variations of  $\epsilon_r$  with frequency for an epoxy-glass substrate.

On the other hand, the relative dielectric constant of the epoxy-glass substrate was found to be dependent upon frequency. [Figure 4](#) shows the variation of  $\epsilon_r$  with frequency for this type of substrate. It was also found that losses in the epoxy-glass board make it impractical for applications above 500 MHz.

As a conclusion, this simplified method can be readily used to determine the relative dielectric constant of a microstrip substrate using a variety of different substrate materials. The approach is based on measuring the input reactance of an open-circuit microstrip line with the aid of a vector network analyzer (VNA) at a specified frequency. The classic microstrip-line equations found in the literature were formulated as a root finder algorithm to search for  $\epsilon_r$ .

## REFERENCES

1. G. Gonzalez, Microwave Transistor Amplifiers Analysis and Design, 2nd ed., Prentice-Hall, Upper Saddle River, NJ, 1997, Chapter 2.

2. I.J. Bahl and D.K. Trivedi, "A Designer's Guide to Microstrip Line" MicroWaves, May 1977, pp.174-182.

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