

One-Port VNA Calibration: A Look Under the Hood

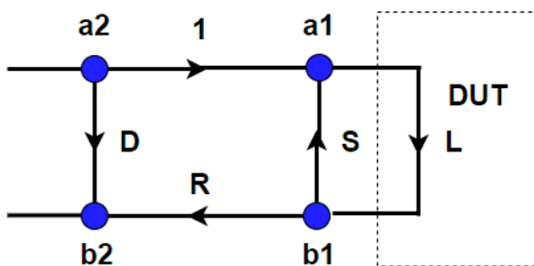
One-port vector-network-analyzer calibration with three-term error correction is relatively easy to understand. This article introduces a convenient matrix formulation that solves for the three error terms.

For a simple one-port vector-network-analyzer (VNA) calibration, only three error terms are needed: directivity (D), source match (S), and reflection tracking (R). These three errors theoretically appear between the VNA and the device under test (DUT). Knowledge of these three error terms allows for correction or calibration of the measurement.

Directivity error is mostly due to leakage in the bridge. Some amount of incident signal causes a small erroneous response on the reflection port. A typical raw bridge directivity might be anywhere from 15 to 40 dB. The source match represents the error in the source impedance of the VNA. The output source match is never perfect and is made somewhat worse by the cable connector and the test cable. A typical raw source match for a VNA might be 20 dB.

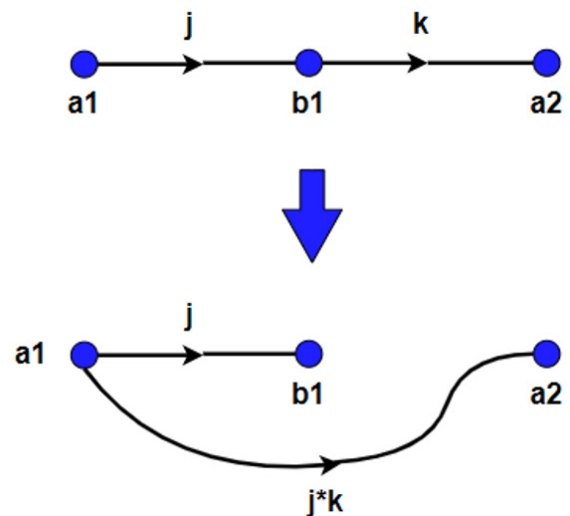
The reflection tracking error is basically the frequency response of the reflected signal from the DUT into the VNA and out through the reflection port of the bridge. This includes the loss incurred in the test cable and looks like a small value at low frequencies, increasing to several decibels at higher frequencies.

Figure 1 depicts a network-signal flow diagram with the



Error Terms

1. Shown is a network-signal flow diagram with the three error terms included.



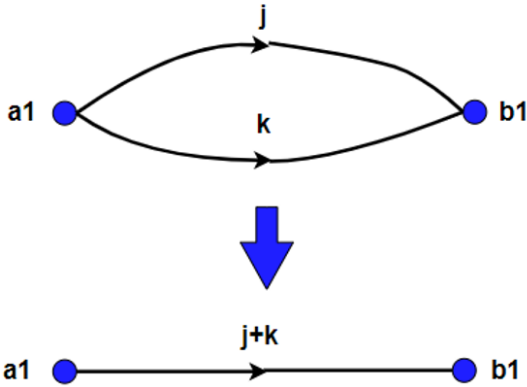
2. This diagram illustrates the series rule: Because $b_1 = j * a_1$ and $a_2 = b_1 * k$, then $a_2 = j * k * a_1$, and one can break out the connection from a_1 to a_2 .

three error terms included. A few simple manipulations of this diagram yield the formula for the measured value as a function of the DUT reflection coefficient and these three error terms.

Note that b_2/a_2 is our measured reflection coefficient and b_1/a_1 is the actual reflection coefficient of L. We want to know b_1/a_1 while measuring b_2/a_2 . This can be simplified with network flow-graph manipulations. For those unfamiliar with the technique, it involves only a few simple rules and an understanding that the math works only in the direction of a flow-graph arrow. For instance, from Figure 1, one can say that $b_2 = R * b_1$. It is NOT true that $b_1 = b_2/R$.

Network Flow-Graph Rules

The first rule is the “Series Rule.” Because $b_1 = j * a_1$ and

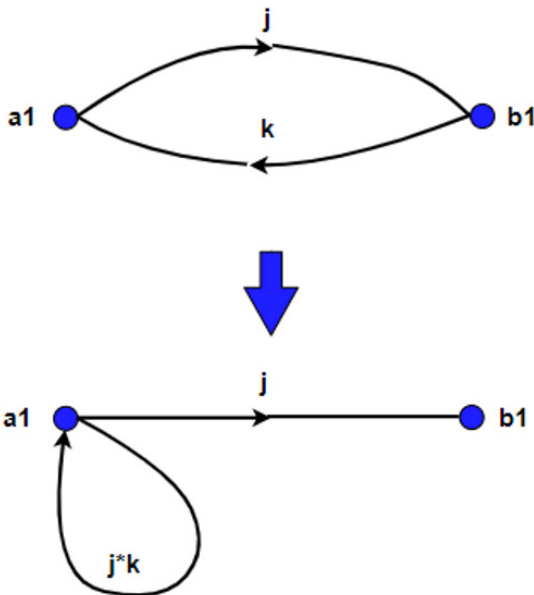


3. Here, we see illustrated the parallel rule: Because both branches “j” and “k” point in the same direction, their contributions may be combined as shown.

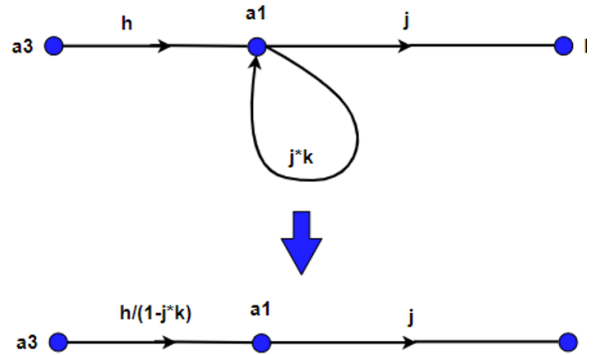
$a_2 = b_1 * k$, then $a_2 = j * k * a_1$ and one can break out the connection from a1 to a2 (Fig. 2). This is useful if a1 is the independent node in the network, as it now gives the other two nodes explicitly.

The next rule is the “Parallel Rule”: Because both branches “j” and “k” point in the same direction, their contributions may be combined as shown in Figure 3.

The next rule is the “Self-Loop Rule”: The loop is created by virtue of the Series Rule as the path from a1 to b1 and back may be combined (Fig. 4). This may not seem like a useful transform, but one more manipulation is possible. A network flow will always enter the loop. The loop may be removed, and its effect applied to that entering path as follows (Fig. 5).



4. According to the self-loop rule, the loop is created by virtue of the series rule as the path from a1 to b1 and back may be combined.



5. Application of the self-loop rule leaves one more possible manipulation: There will always be a network flow that enters the loop. The loop may be removed, and its effect applied to that entering path is shown.

This transform comes from the parallel rule:

$$a_1 = h * a_3 + j * k * a_1$$

which reduces to:

$$a_1 = \frac{h}{1-j*k} * a_3$$

If the arrow from b1 pointed to the left and entered the loop, then “j” would have to be replaced by:

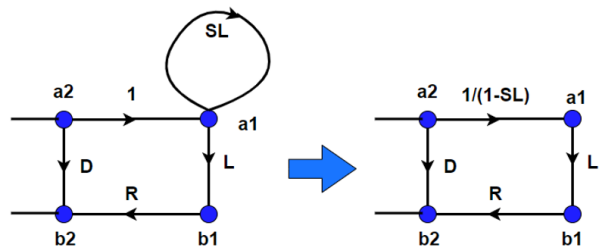
$$\frac{j}{1-j*k}$$

These rules may now be employed to simplify Figure 1 such that all nodes are explicitly defined by the single independent node “a2.” The steps encompass the transformation depicted in Figure 6, followed by the application of series and parallel rules as shown in Figure 7.

Finally, we see that our measured reflection coefficient is represented by:

$$\Gamma_m = \frac{b_2}{a_2} = D + \frac{RL}{1-SL}$$

We can solve this for L and obtain:



6. These rules may now be employed to simplify Figure 1 such that all nodes are explicitly defined by the single independent node “a2.” First, as shown, the self-loop rule is applied to eliminate a branch.



7. Finally, the series rule is applied, followed by the parallel rule, to arrive at the simplification of Figure 1.

$$L = \frac{\Gamma_m - D}{R + S(\Gamma_m - D)}$$

which is the result we were looking for.

Now, if we know D, R, and S, we can easily correct our Γ_m to obtain L, which is the actual reflection coefficient.

One-Port Calibration

We now make three measurements of three known artifacts and solve a system of equations to obtain the D, R, and S error terms. In addition, we could make more than three measurements and perhaps improve our estimation of the error terms with a least-squares approach. Rather than write out those three equations, we'll jump right to the matrix notation, which is much cleaner.

Form two matrices, C and V:

$$C = \begin{bmatrix} \Gamma_{a1} & 1 & \Gamma_{a1}\Gamma_{m1} \\ \Gamma_{a2} & 1 & \Gamma_{a2}\Gamma_{m2} \\ \Gamma_{a3} & 1 & \Gamma_{a3}\Gamma_{m3} \end{bmatrix}$$

and

$$V = \begin{bmatrix} \Gamma_{m1} \\ \Gamma_{m2} \\ \Gamma_{m3} \end{bmatrix}$$

where the Γ_a values are the actual values and the Γ_m values are the measured values. We must know these actual values a priori. These could be an open, short, and load, where the actual values are characterized by short delays and parasitic capacitance or inductance as is done in a calibration kit. The three actual values could also be three shorts with different delays such that the three reflection coefficients are spread around the outside of the Smith chart over frequency.

Any three artifacts with known reflection coefficients may be used if they're sufficiently separated on the Smith chart at every frequency. If they're not, the matrix calculations will be ill-conditioned and the results unreliable.

With these two matrices, calculate matrix E:

$$E = (C^H * C)^{-1} * C^H * V = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

where H is the Hermetian transpose operator, or the matrix transposed with its entries conjugated.

From E, we can find D, R, and S: $D = E_2$, $S = E_3$, and $R = E_1 + E_2 * E_3$

It so happens that $(C^H * C)^{-1} * C^H$ is a least-squares calculation. If our measurements are a little noisy, we can improve our results by making more known measurements. Simply add more rows to the C and V matrices. Matrix E will still have three values in the end and the results might be somewhat better in the face of slightly noisy measurements.

Conclusion

In this article, we introduced the three-term error-correction terms, and used a simple method of derivation to show how these error terms affect the measured value. Finally, we outlined a simple matrix method for easy calculation of one-port error correction to arrive at calibrated results from the error terms. None of this work is original, but it's educational to pull all of the pieces together and *demonstrate how they are used*.

At Copper Mountain Technologies, Brian Walker is the Senior RF Engineer helping customers to resolve technical issues and working to develop new solutions for applications of VNAs in test and measurement. Previously, he was manager of RF design at Bird Electronics, where he led a team of RF designers in creating new and innovative products. Prior to that, he worked for Motorola Component Products Group and was responsible for the design of ceramic comb-line filters for communications devices. Brian graduated from the University of New Mexico, has 40 years of RF design experience, and has authored three U.S. patents.

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